Congressional Research Service

Report RL31074

# *The House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States*


David C. Huckabee, Government and Finance Division



Updated August 10, 2001



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Order Code RL31074

# CRS Report for Congress

Received through the CRS Web

# The House of Representatives Apportionment Formula: An Analysis of Proposals for Change and Their Impact on States

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## Summary

Now that the reallocation of Representatives among the states based on the 2000 Census has been completed, some members of the statistical community are urging Congress to consider changing the current House apportionment formula. However, other formulas also raise questions.

Seats in the House of Representatives are allocated by a formula known as the Hill, or equal proportions, method. If Congress decided to change it, there are at least five alternatives to consider. Four of these are based on *rounding* fractions; one, on *ranking* fractions. The current apportionment system (codified in 2 U.S.C. 2a) is one of the rounding methods.

The Hamilton-Vinton method is based on ranking fractions. First, the population of 50 states is divided by 435 (the House size) in order to find the national "ideal size" district. Next this number is divided into each state's population. Each state is then awarded the whole number in its quotient (but at least one). If fewer than 435 seats have been assigned by this process, the fractional remainders of the 50 states are rank-ordered from largest to smallest, and seats are assigned in this manner until 435 are allocated.

The *rounding* methods, including the Hill method currently in use, allocate seats among the states differently, but operationally the methods only differ by where rounding occurs in seat assignments. Three of these methods — Adams, Webster, and Jefferson — have fixed rounding points. Two others — Dean and Hill — use varying rounding points that rise as the number of seats assigned to a state grows larger. The methods can be defined in the same way (after substituting the appropriate rounding principle in parentheses). The rounding point for Adams is (*up for all fractions*); for Dean (*at the harmonic mean*); for Hill (*at the geometric mean*); for Webster (*at the arithmetic mean — .5*); and for Jefferson (*down for all fractions*). Substitute these phrases in the general definition below for the rounding methods:

> *Find a number so that when it is divided into each state's population and resulting quotients are rounded (substitute appropriate phrase), the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional requirement.)*

Unlike the Hamilton-Vinton method, which uses the national "ideal size" district for a divisor, the rounding methods use a *sliding divisor*. If the national "ideal size" district results in a 435-seat House after rounding according to the rule of method, no alteration in its size is necessary. If too many seats are allocated, the divisor is made larger (it *slides* up); if too few seats are apportioned, the divisor becomes smaller (it *slides* down). Fundamental to choosing an apportionment method is a determination of fairness. Each of the competing formulas is the best method for satisfying one or more mathematical tests.

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## Introduction

Now that the reallocation of Representatives among the states based on the 2000 Census has been completed, some members of the statistical community are urging Congress to consider changing the current House apportionment formula. However, other formulas also raise questions.[2]

In 1991, the reapportionment of the House of Representatives was nearly overturned because the current "equal proportions" formula for the House apportionment was held to be unconstitutional by a three-judge federal district court. The court concluded that:

> By complacently relying, for over fifty years, on an apportionment method which does not even consider absolute population variances between districts, Congress has ignored the goal of equal representation for equal numbers of people. The court finds that unjustified and avoidable population differences between districts exist under the present apportionment, and ... [declares] section 2a of Title 2, United States Code unconstitutional and void.[3]

The three-judge panel's decision came almost on the 50th anniversary of the current formula's enactment.[4]

The government appealed the panel's decision to the Supreme Court, where Montana argued that the equal proportions formula violated the Constitution because it "does not achieve the greatest possible equality in number of individuals per Representative." This reasoning did not prevail, because, as Justice Stevens wrote in his opinion for a unanimous court, absolute and relative differences in district sizes

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[1] This report originally was authored by David C. Huckabee, who has retired from CRS.

[2] See: Brookings Institution Policy Brief, *Dividing the House: Why Congress Should Reinstate the Old Reapportionment Formula*, by H. Peyton Young, Policy Brief No. 88 (Washington, Brookings Institution, August 2001). Young suggests that Congress consider the matter "now — well in advance of the next census," p. 1.

[3] *Montana v. Department of Commerce*, No. CV. 91-22-H-CCL.(D. Mt. Oct. 18, 1991). U.S. District Court for the District of Montana, Helena Division.

[4] 55 *Stat.* 761, codified in 2 U.S.C. 2a, was enacted November 15, 1941.

are identical when considering deviations in district populations *within* states, but they are different when comparing district populations *among* states. Justice Stevens noted, however, that "although common sense" supports a test requiring a "good faith effort to achieve precise mathematical equality *within* each State ... the constraints imposed by Article I, §2, itself make that goal illusory for the nation as a whole." He concluded "that Congress had ample power to enact the statutory procedure in 1941 and to apply the method of equal proportions after the 1990 census."[5]

The year 1991 was a banner year for court challenges on the apportionment front. At the same time the Montana case was being argued, another case was being litigated by Massachusetts. The Bay State lost a seat to Washington because of the inclusion of 978,819 federal employees stationed overseas in the state populations used to determine reapportionment. The court ruled that Massachusetts could not challenge the President's decision to include the overseas federal employees in the apportionment counts, in part because the President is not subject to the terms of the Administrative Procedures Act.[6]

In 2001, the Census Bureau's decision to again include the overseas federal employees in the population used to reapportion the House produced a new challenge to the apportionment population. Utah argued that it lost a congressional seat to North Carolina because of the Bureau's decision to include overseas federal employees in the apportionment count, but not other citizens living abroad. Utah said that Mormon missionaries were absent from the state because they were on assignment: a status similar to federal employees stationed overseas. Thus, the state argued, the Census Bureau should have included the missionaries in Utah's apportionment count. The state further argued that, unlike other U.S. citizens living overseas, missionaries could be accurately reallocated to their home states because the Mormon church has excellent administrative records. Utah's complaint was dismissed by a three-judge federal court on April 17, 2001.[7]

The Supreme Court appears to have settled the issue about Congress's discretion to choose a method to apportion the House, and has granted broad discretion to the President in determining who should be included in the population used to allocate seats. Although modern Congresses have rarely considered the issue of the formula

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[5] *Department of Commerce* v. *Montana* 503 U.S. 442 (1992).

[6] *Franklin* v. *Massachusetts,* 505 U.S. 788 (1992). The Administrative Procedures Act (APA) sets forth the procedures by which federal agencies are accountable to the public and their actions are subject to review by the courts. Since the Supreme Court ruled that a President's decisions are not subject to review under the APA by courts, the district court's decision to the contrary was reversed. Plaintiffs in this case also challenged the House apportionment formula, arguing that the Hill (equal proportions) method discriminated against larger states.

[7] *Utah* v. *Evans*, No. F-2-01-CV-23: B (D. Utah, complaint filed Jan. 10, 2000). Representative Gilman introduced H.R. 1745, the Full Equality for Americans Abroad Act, on May 8, 2001. The bill would require including all citizens living abroad in the state populations used for future apportionments. For further reading on this and other legal matters pertaining to the 2000 census, see CRS Report RL30870, *Census 2000: Legal Issues re: Data for Reapportionment and Redistricting*, by Margaret Mikyung Lee.

used in the calculations, this report describes apportionment options from which Congress could choose and the criteria that each method satisfies.[8]

## Background

One of the fundamental issues before the framers at the constitutional convention in 1787 was how power was to be allocated in Congress between the smaller and larger states. The solution ultimately adopted became known as the Great (or Connecticut) Compromise. It solved the controversy between large and small states by creating a bicameral Congress with states equally represented in the Senate and seats allocated by population in the House. The Constitution provided the first apportionment: 65 Representatives were allocated to the states based on the framers' estimates of how seats might be apportioned after a census. House apportionments thereafter were to be based on Article 1, section 2, as modified by clause 2 of the Fourteenth Amendment:

> *Amendment XIV, section 2.* Representatives and direct taxes shall be apportioned among the several States ... according to their respective numbers ....
>
> *Article 1, section 2.* The number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at least one Representative ....

The constitutional mandate that Representatives would be apportioned according to population did not describe how Congress was to distribute fractional entitlements to Representatives. Clearly there would be fractions because districts could not cross state lines and the states' populations were unlikely to be evenly divisible. From its beginning in 1789 Congress was faced with deciding how to apportion the House of Representatives. The controversy continued until 1941, with the enactment of the Hill ("equal proportions") method. During congressional debates on apportionment, the major issues were how populous a congressional district ought to be (later re-cast as how large the House ought to be), and how fractional entitlements to Representatives should be treated. The matter of the permanent House size has received little attention since it was last increased to 435 after the 1910 Census.[9] The Montana legal challenge added a new perspective to the picture — determining which method comes closest to meeting the goal of "one person, one vote."

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[8] Representative Fithian (H.R. 1990) and Senator Lugar (S. 695) introduced bills in the 97th Congress to adopt the Hamilton-Vinton method of apportionment to be effective for the 1980 and subsequent censuses. Hearings were held in the House, but no further action was taken.

[9] Article I, Section 3 defines both the maximum and minimum size of the House; the actual House size is set by law. There can be no fewer than one Representative per state, and no more than one for every 30,000 persons. Thus, the House after 2001 could have been as small as 50 and as large as 9,361 Representatives (30,000 divided into the total U.S. apportionment population).

The "one person, one vote" concept was established through a series of Supreme Court decisions beginning in the 1960s. The court ruled in 1962 that state legislative districts must be approximately equal in population (*Baker v. Carr,* 369 U.S. 186). This ruling was extended to the U.S. House of Representatives in 1964 (*Wessberry v. Sanders,* 376 U.S. 1). Thus far, the concept has only been applied *within* states. states must be able to justify *any* deviations from absolute numerical equality for their congressional districts in order to comply with a 1983 Supreme Court decision — *Karcher v. Daggett* (462 U.S. 725).

The population distribution among states in the 2000 Census, combined with a House size of 435, and the requirement that districts not cross state lines, means that there is a wide disparity in district sizes — from 495,304 (Wyoming) to 905,316 (Montana) after the 2000 Census. This *interstate* population disparity among districts in 2001 contrasts with the *intrastate* variation experienced in the redistrictings following the 1990 Census. Nineteen of the 43 states that had two or more districts in 1992 drew districts with a population difference between their districts of ten persons or fewer, and only six states varied by more than 1,000 persons.[10]

Given a fixed-size House and an increasing population, there will inevitably be population deviations in district sizes among states; what should be the goal of an apportionment method? Although Daniel Webster was a proponent of a particular formula (the major fractions method), he succinctly defined the apportionment problem during debate on an apportionment bill in 1832. Webster said that:

> The Constitution, therefore, must be understood, not as enjoining an absolute relative equality, because that would be demanding an impossibility, but as requiring of Congress to make the apportionment of Representatives among the several states according to their respective numbers, as near as may be. That which cannot be done perfectly must be done in a manner as near perfection as can be ....[11]

Which apportionment method is the "manner as near perfection as can be"? Although there are potentially thousands of different ways in which the House can be apportioned, six methods are most often mentioned as possibilities. These are the methods of: Hamilton-Vinton, "largest fractional remainders"; Adams, "smallest divisors"; Dean, "harmonic mean"; Hill, "equal proportions"; Webster, "major fractions"; and Jefferson, "largest divisors."

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[10] CRS Report 93-1060 GOV, *Congressional Redistricting: Federal Law Controls a State Process,* by Royce Crocker, pp. 53-54.

[11] M. L. Balinski and H. P. Young, *Fair Representation*, 2nd ed. (Washington: Brookings Institution Press, 2001), p. 31.

# Apportionment Methods Defined

## Hamilton-Vinton: Ranking Fractional Remainders

Why is there a controversy? Why not apportion the House the intuitive way by dividing each state's population by the national "ideal size" district (645,632 in 2001) and give each state its "quota" (rounding up at fractional remainders of .5 and above, and down for remainders less than .5)? The problem with this proposal is that the House size would fluctuate around 435 seats. In some decades, the House might include 435 seats; in others, it might be either under or over the legal limit. In 2001, this method would result in a 433-seat House (438 in 1991).

One solution to this problem of too few or too many seats would be to divide each state's population by the national "ideal" size district, but instead of rounding at the .5 point, allot each state initially the whole number of seats in its quota (except that states entitled to less than one seat would receive one regardless). Next, rank the fractional remainders of the quotas in order from largest to smallest. Finally, assign seats in rank order until 435 are allocated (see Table 1). If this system had been used in 2001, California would have one less Representative, and Utah would have one more.

This apportionment formula, which is associated with Alexander Hamilton, was used in Congress's first effort to enact an apportionment of the House. The bill was vetoed by President Washington — his first exercise of this power.[12] This procedure, which might be described as the largest fractional remainders method, was used by Congress from 1851 to 1901;[13] but it was never strictly followed because changes were made in the apportionments that were not consistent with the method.[14] It has generally been known as the Vinton method (for Representative Samuel Vinton (Ohio), its chief proponent after the 1850 Census). Assuming a fixed House size, the Hamilton-Vinton method can be described as follows:

*Hamilton-Vinton*

> *Divide the apportionment population[15] by the size of the House to obtain the "ideal congressional district size" to be used as a divisor. Divide each state's population by the ideal size district to obtain its quota. Award each state the whole number obtained in these quotas. (If a state receives less than one Representative, it automatically receives one because of the constitutional requirement.) If the number of Representatives assigned using the whole numbers is less than the House total, rank the fractional*

[12] *Fair Representation*, p. 21.

[13] Laurence F. Schmeckebier, *Congressional Apportionment* (Washington: The Brookings Institution, 1941). p. 73.

[14] *Fair Representation*, p. 37.

[15] The apportionment population is the population of the fifty states found by the Census.

*remainders of the states' quotas and award seats in rank order from highest to lowest until the House size is reached.*

The Hamilton-Vinton method has simplicity in its favor, but its downfall was the *Alabama paradox*. Although the phenomenon had been observed previously, the "paradox" became an issue after the 1880 census when C. W. Seaton, Chief Clerk of the Census Office, wrote the Congress on October 25, 1881, stating:

> While making these calculations I met with the so-called "Alabama" paradox where Alabama was allotted 8 Representatives out of a total of 299, receiving but 7 when the total became 300.[16]

Alabama's loss of its eighth seat when the House size was increased resulted from the vagaries of fractional remainders. With 299 seats, Alabama's quota was 7.646 seats. It was allocated eight seats based on this quota, but it was on the dividing point. When a House size of 300 was used, Alabama's quota increased to 7.671, but Illinois and Texas now had larger fractional remainders than Alabama. Accordingly, each received an additional seat in the allotment of fractional remainders, but since the House had increased in size by only one seat, Alabama lost the seat it had received in the allotment by fractional remainders for 299 seats.[17] This property of the Hamilton-Vinton method became a big enough issue that the formula was changed in 1911.

One could argue that the Alabama paradox should not be an important consideration in apportionments, since the House size was fixed in size at 435, but the Hamilton-Vinton method is subject to other anomalies. Hamilton-Vinton is also subject to the *population paradox* and the *new states paradox.*

The population paradox occurs when a state that grows at a greater percentage rate than another has to give up a seat to the slower growing state. The new states paradox works in much the same way — at the next apportionment after a new state enters the Union, any increase in House size caused by the additional seats for the new state may result in seat shifts among states that otherwise would not have happened. Finding a formula that avoided the paradoxes was a goal when Congress adopted a rounding, rather than a ranking, method when the apportionment law was changed in 1911.

**Table 1** illustrates how a Hamilton-Vinton apportionment would be done by ranking the fractional remainders of the state's quotas in order from largest to smallest. In 2001 North Carolina and Utah's fractional remainders of less than 0.5 would have been rounded up by the Hamilton-Vinton method in order for the House to have totaled 435 Representatives.

[16] *Fair Representation*, p. 38.

[17] Ibid., p. 39.

## Table 1. Apportioning the House in 2001 by Simple Rounding and Ranked Fractional Remainders (Hamilton-Vinton)

| States ranked by fractional remainders | Quota | Whole number of seats assigned | Fractional remainders | Allocation of seats | |
|---|---|---|---|---|---|
| | | | | Hamilton-Vinton | Simple rounding |
| North Dakota | 0.995 | 1 | 0.99506 | 1 | 1 |
| Virginia | 10.976 | 10 | 0.97562 | 11 | 11 |
| Maine | 1.975 | 1 | 0.97500 | 2 | 2 |
| Alaska | 0.972 | 1 | 0.97215 | 1 | 1 |
| Arizona | 7.946 | 7 | 0.94600 | 8 | 8 |
| Vermont | 0.943 | 1 | 0.94271 | 1 | 1 |
| Louisiana | 6.925 | 6 | 0.92520 | 7 | 7 |
| New Hampshire | 1.914 | 1 | 0.91423 | 2 | 2 |
| Alabama | 6.896 | 6 | 0.89561 | 7 | 7 |
| Hawaii | 1.881 | 1 | 0.88058 | 2 | 2 |
| Massachusetts | 9.824 | 9 | 0.82386 | 10 | 10 |
| New Mexico | 2.819 | 2 | 0.81910 | 3 | 3 |
| Tennessee | 8.811 | 8 | 0.81060 | 9 | 9 |
| West Virginia | 2.802 | 2 | 0.80249 | 3 | 3 |
| Florida | 24.776 | 24 | 0.77601 | 25 | 25 |
| Wyoming | 0.766 | 1 | 0.76560 | 1 | 1 |
| Georgia | 12.686 | 12 | 0.68560 | 13 | 13 |
| Missouri | 8.666 | 8 | 0.66565 | 9 | 9 |
| Colorado | 6.665 | 6 | 0.66492 | 7 | 7 |
| Nebraska | 2.651 | 2 | 0.65146 | 3 | 3 |
| Rhode Island | 1.622 | 1 | 0.62247 | 2 | 2 |
| Minnesota | 7.614 | 7 | 0.61365 | 8 | 8 |
| Ohio | 17.582 | 17 | 0.58173 | 18 | 18 |
| Iowa | 4.532 | 4 | 0.53190 | 5 | 5 |
| **North Carolina** | **12.470** | **12** | **0.47028** | **13** | **12** |
| **Utah** | **3.457** | **3** | **0.45731** | **4** | **3** |
| California | 52.447 | 52 | 0.44715 | 52 | 52 |
| Indiana | 9.415 | 9 | 0.41458 | 9 | 9 |
| Mississippi | 4.410 | 4 | 0.40980 | 4 | 4 |
| Montana | 1.399 | 1 | 0.39936 | 1 | 1 |
| Michigan | 15.389 | 15 | 0.38882 | 15 | 15 |
| New York | 29.376 | 29 | 0.37617 | 29 | 29 |
| Oklahoma | 5.346 | 5 | 0.34633 | 5 | 5 |
| Texas | 32.312 | 32 | 0.31150 | 32 | 32 |
| Wisconsin | 8.302 | 8 | 0.30233 | 8 | 8 |
| Oregon | 5.300 | 5 | 0.29953 | 5 | 5 |
| Connecticut | 5.270 | 5 | 0.27015 | 5 | 5 |
| Kentucky | 6.259 | 6 | 0.25924 | 6 | 6 |
| Illinois | 19.227 | 19 | 0.22714 | 19 | 19 |
| South Carolina | 6.222 | 6 | 0.22157 | 6 | 6 |
| Delaware | 1.213 | 1 | 0.21349 | 1 | 1 |
| Maryland | 8.204 | 8 | 0.20445 | 8 | 8 |
| South Dakota | 1.170 | 1 | 0.16991 | 1 | 1 |
| Kansas | 4.164 | 4 | 0.16387 | 4 | 4 |
| Arkansas | 4.142 | 4 | 0.14209 | 4 | 4 |
| Washington | 9.133 | 9 | 0.13311 | 9 | 9 |

| States ranked by fractional remainders | Quota | Whole number of seats assigned | Fractional remainders | Allocation of seats | |
|---|---|---|---|---|---|
| | | | | Hamilton-Vinton | Simple rounding |
| Nevada | 3.095 | 3 | 0.09456 | 3 | 3 |
| New Jersey | 13.022 | 13 | 0.02160 | 13 | 13 |
| Pennsylvania | 19.013 | 19 | 0.01326 | 19 | 19 |
| Idaho | 2.005 | 2 | 0.00521 | 2 | 2 |
| Total | 435 | 413 | | 435 | 433 |

Source: Data calculated by CRS. The "quota" is found by dividing the state population by the national "ideal size" district (645,632 based on the 2000 Census). North Carolina and Utah receive additional seats with the Hamilton-Vinton system even though their fractional remainders are less than .5.

## Rounding Methods

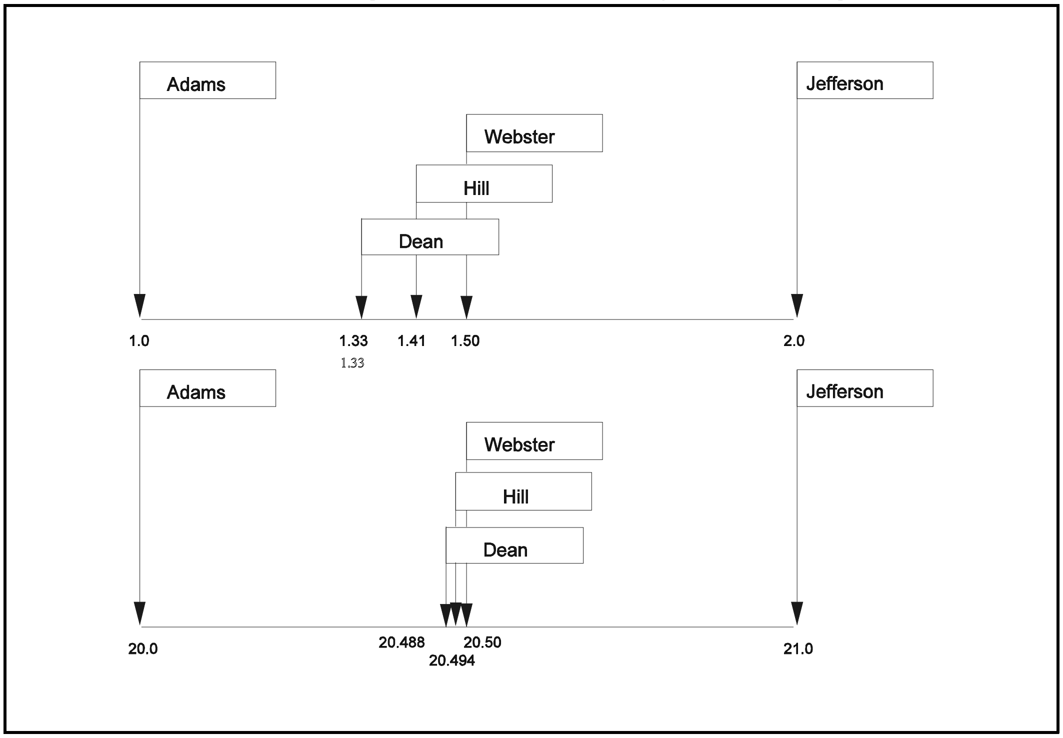
The kinds of calculations required by the Hamilton-Vinton method are paralleled, in their essentials, in all the alternative methods that are most frequently discussed — but fractional remainders are rounded instead of ranked. First, the total *apportionment population*, (the population of the 50 states as found by the census) is divided by 435, or the size of the House. This calculation yields the national "ideal" district size. Second, the "ideal" district size is used as a common divisor for the population of each state, yielding what are called the states' *quotas* of Representatives. Because the quotas still contain fractional remainders, each method then obtains its final apportionment by rounding its allotments either up or down to the nearest whole number according to certain rules.

The operational difference between the methods lies in how each defines the *rounding point* for the fractional remainders in the allotments — that is, the point at which the fractions rounded down are separated from those rounded up. Each of the rounding methods defines its rounding point in terms of some mathematical quantity. Above this specified figure, all fractional remainders are automatically rounded up; those below, are rounded down.

For a given common divisor, therefore, each rounding method yields a set number of seats. If using national "ideal" district size as the common divisor results in 435 seats being allocated, no further adjustment of the divisor is necessary. But if too many or too few seats are apportioned, the common divisor must be varied until a value is found that yields the desired number of seats. (These methods will, as a result, generate allocations before rounding that differ from the states' quotas.) If too many seats are apportioned, a larger divisor is tried (the divisor *slides* up); if too few, a smaller divisor (it *slides* down). The divisor finally used is that which apportions of a number of seats equal to the desired size of the House.[18]

[18] Balinski and Young, in *Fair Representation*, refer to these as *divisor* methods because they use a common divisor. This report characterizes them as *rounding* methods, although they use common divisors, because the Hamilton-Vinton method also uses a common divisor, while its actual apportionment is not based on rounding. All these methods can be described in different ways, but looking at them based on how they treat quotients provides (continued...)

**Figure 1. Illustrative Rounding Points for Five Apportionment Methods (for Two and Twenty-one Seats)**




This illustration is adapted from, Balinski, M. L. and H. P. Young, *Fair Representation*, 2nd ed. (Washington: Brookings Institution Press, 2001), pp. 63-65.

The rounding methods that are mentioned most often (although there could be many more) are the methods of: Webster ("major fractions"); Hill ("equal proportions" — the current method); Dean ("harmonic mean"); Adams ("smallest divisors"); and Jefferson ("greatest divisors"). Under any of these methods, the Census Bureau would construct a priority list of claims to representation in the House.[19] The key difference among these methods is in the rule by which the rounding point is set — that is, the rule that determines what fractional remainders result in a state being rounded up, rather than down.

In the Adams, Webster, and Jefferson methods, the rounding points used are the same for a state of any size. In the Dean and Hill methods, on the other hand, the rounding point varies with the number of seats assigned to the state; it rises as the the state's population increases. With these two methods, in other words, smaller (less populous) states will have their apportionments rounded up to yield an extra seat for smaller fractional remainders than will larger states. This property provides

[18] (...continued)
a consistent framework to understand them all.

[19] For a detailed explanation of how apportionments are done using priority lists, see CRS Report RL30711, *The House Apportionment Formula in Theory and Practice*, by Royce Crocker.

the intuitive basis for challenging the Dean and Hill methods as favoring small states at the expense of the large (more populous) states.[20]

These differences among the rounding methods are illustrated in **Figure 1**. The "flags" in Figure 1 indicate the points that a state's fractional remainder must exceed for it to receive a second seat, and to receive a 21st seat. Figure 1 visually illustrates that the only rounding points which change their relative positions are those for Dean and Hill. Using the rounding points for a second seat as the example, the Adams method awards a second seat for any fractional remainder above one. Dean awards the second seat for any fractional remainder above 1.33. Similarly, Hill gives a second seat for every fraction exceeding 1.41, Webster, 1.5, and Jefferson does not give a second seat until its integer value of a state's quotient equals or exceeds two.

**Webster: Rounding at the Midpoint (.5).** The easiest rounding method to describe is the Webster ("major fractions") method which allocates seats by rounding up to the next seat when a state has a remainder of .5 and above. In other words, it rounds fractions to the lower or next higher whole number at the *arithmetic mean*, which is the midpoint between numbers. For example, between 1 and 2 the arithmetic mean is 1.5; between 2 and 3, the arithmetic mean is 2.5, etc. The Webster method (which was used in 1840, 1910, and 1930) can be defined in the following manner for a 435-seat House:

*Webster*

> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the arithmetic mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

**Hill: Rounding at the Geometric Mean.** The only operational difference between a Webster and a Hill apportionment (equal proportions — the method in use since 1941), is where the rounding occurs. Rather than rounding at the *arithmetic mean* between the next lower and the next higher whole number, Hill rounds at the *geometric mean.* The geometric mean is the square root of the multiplication of two numbers. The Hill rounding point between 1 and 2, for example, is 1.41 (the square root of 2), rather than 1.5. The rounding point between 10 and 11 is the square root of 110, or 10.487. The Hill method can be defined in the following manner for a 435-seat House:

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[20] Peyton Young states that the Hill method "systematically favors the small states by 3-4 percent." He determined this figure by first eliminating from the calculations the very small states whose quotas equaled less than one half a Representative. He then computed the relative bias for the methods described in this report for all the censuses based on the "per capita representation in the large states as a group and in the small states as group. The percentage difference between the two is the method's relative bias toward small states in that year. To estimate their long-run behavior, I compute the average bias of each method up to that point in time." See: Brookings Institution Policy Brief No. 88, *Dividing the House: Why Congress Should Reinstate the Old Reapportionment Formula*, p. 4.

*Hill*

> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the geometric mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

**Dean: Rounding at the Harmonic Mean.** The Dean method (advocated by Montana) rounds at a different point — the *harmonic mean* between consecutive numbers. The harmonic mean is obtained by multiplying the product of two numbers by 2, and then dividing that product by the sum of the two numbers.[21] The Dean rounding point between 1 and 2, for example, is 1.33, rather than 1.5. The rounding point between 10 and 11 is 10.476. The Dean method (which has never been used) can be defined in the following manner for a 435-seat House:

*Dean*

> *Find a number so that when it is divided into each state's population and resulting quotients are rounded at the harmonic mean, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

**Adams: All Fractions Rounded Up.** The Adams method ("smallest divisors") rounds up to the next seat for any fractional remainder. The rounding point between 1 and 2, for example, would be any fraction exceeding 1 with similar rounding points for all other integers. The Adams method (which has never been used, but is also advocated by Montana) can be defined in the following manner for a 435-seat House:

*Adams*

> *Find a number so that when it is divided into each state's population and resulting quotients that include fractions are rounded up, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional entitlement.)*

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[21] Expressed as a formula, the harmonic mean ($H$) of the numbers ($A$) and ($B$) is: $H = 2*(A*B)/(A+B)$.

**Jefferson: All Fractions Rounded Down.** The Jefferson method ("largest divisors") rounds down any fractional remainder. In order to receive 2 seats, for example, a state would need 2 in its quotient, but it would not get 3 seats until it had 3 in its quotient. The Jefferson method (used from 1790 to 1830) can be defined in the following manner for a 435-seat House:

*Jefferson*

> *Find a number so that when it is divided into each state's population and resulting quotients that include fractions are rounded down, the total number of seats will sum to 435. (In all cases where a state would be entitled to less than one seat, it receives one anyway because of the constitutional requirement.)*

## Changing the Formula: The Impact in 2001

What would happen in 2001 if any of the alternative formulas discussed in this report were to be adopted? As compared to the Hill (equal proportions) apportionment currently mandated by law, the Dean method, advocated by Montana in 1991, results (not surprisingly) in Montana regaining its second seat that it lost in 1991, and Utah gaining a fourth seat. Neither California nor North Carolina would have gained seats in 2001 using the Dean method. The Webster method would have caused no change in 2001, but in 1991 it would have resulted in Massachusetts retaining a seat it would otherwise would have lost under Hill, while Oklahoma would have lost a seat. The Hamilton-Vinton method (as discussed earlier) results in Utah gaining and California not gaining a seat as compared to the current (Hill) method. The Adams method in 2001 would reassign eight seats among fourteen states (see Table 2). The Jefferson method would reassign six seats among twelve states (see Table 2).

**Tables 2 and 3**, which follow, present seat allocations based on the 2000 Census for the six methods discussed in this report. Table 2 is arranged in alphabetical order. Table 3 is arranged by total state population, rank-ordered from the most populous state (California) to the least (Wyoming). This table facilitates evaluating apportionment methods by looking at their impact according to the size of the states. Allocations that differ from the current method are ***bolded and italicized*** in both tables.

## Table 2. Seat Assignments in 2001 for Various House Apportionment Formulas (Alphabetical Order)

| ST | Apportion-ment population | Quota[a] | Apportionment Method: Smallest divisors (Adams) | Harm-onic mean (Dean) | Ranked fractional remainders (Hamilton-Vinton) | **Current method: equal pro-portions (Hill)** | Major fractions (Webster) | Largest divisors (Jeffer-son) |
|---|---|---|---|---|---|---|---|---|
| AL | 4,461,130 | 6.896 | 7 | 7 | 7 | 7 | 7 | 7 |
| AK | 628,933 | 0.972 | 1 | 1 | 1 | 1 | 1 | 1 |
| AZ | 5,140,683 | 7.946 | 8 | 8 | 8 | 8 | 8 | 8 |
| AR | 2,679,733 | 4.142 | 4 | 4 | 4 | 4 | 4 | 4 |
| CA | 33,930,798 | 52.447 | ***50*** | ***52*** | ***52*** | 53 | 53 | ***55*** |
| CO | 4,311,882 | 6.665 | 7 | 7 | 7 | 7 | 7 | 7 |
| CT | 3,409,535 | 5.270 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| DE | 785,068 | 1.213 | ***2*** | 1 | 1 | 1 | 1 | 1 |
| FL | 16,028,890 | 24.776 | ***24*** | 25 | 25 | 25 | 25 | ***26*** |
| GA | 8,206,975 | 12.686 | 13 | 13 | 13 | 13 | 13 | 13 |
| HI | 1,216,642 | 1.881 | 2 | 2 | 2 | 2 | 2 | ***1*** |
| ID | 1,297,274 | 2.005 | 2 | 2 | 2 | 2 | 2 | 2 |
| IL | 12,439,042 | 19.227 | 19 | 19 | 19 | 19 | 19 | ***20*** |
| IN | 6,090,782 | 9.415 | 9 | 9 | 9 | 9 | 9 | 9 |
| IA | 2,931,923 | 4.532 | 5 | 5 | 5 | 5 | 5 | ***4*** |
| KS | 2,693,824 | 4.164 | 4 | 4 | 4 | 4 | 4 | 4 |
| KY | 4,049,431 | 6.259 | 6 | 6 | 6 | 6 | 6 | 6 |
| LA | 4,480,271 | 6.925 | 7 | 7 | 7 | 7 | 7 | 7 |
| ME | 1,277,731 | 1.975 | 2 | 2 | 2 | 2 | 2 | 2 |
| MD | 5,307,886 | 8.204 | 8 | 8 | 8 | 8 | 8 | 8 |
| MA | 6,355,568 | 9.824 | 10 | 10 | 10 | 10 | 10 | 10 |
| MI | 9,955,829 | 15.389 | 15 | 15 | 15 | 15 | 15 | ***16*** |
| MN | 4,925,670 | 7.614 | 8 | 8 | 8 | 8 | 8 | ***7*** |
| MS | 2,852,927 | 4.410 | ***5*** | 4 | 4 | 4 | 4 | 4 |
| MO | 5,606,260 | 8.666 | 9 | 9 | 9 | 9 | 9 | 9 |
| MT | 905,316 | 1.399 | ***2*** | ***2*** | 1 | 1 | 1 | 1 |
| NE | 1,715,369 | 2.651 | 3 | 3 | 3 | 3 | 3 | 2 |
| NV | 2,002,032 | 3.095 | 3 | 3 | 3 | 3 | 3 | 3 |
| NH | 1,238,415 | 1.914 | 2 | 2 | 2 | 2 | 2 | 2 |
| NJ | 8,424,354 | 13.022 | 13 | 13 | 13 | 13 | 13 | 13 |
| NM | 1,823,821 | 2.819 | 3 | 3 | 3 | 3 | 3 | ***2*** |
| NY | 19,004,973 | 29.376 | ***28*** | 29 | 29 | 29 | 29 | ***30*** |
| NC | 8,067,673 | 12.470 | ***12*** | ***12*** | 13 | 13 | 13 | 13 |
| ND | 643,756 | 0.995 | 1 | 1 | 1 | 1 | 1 | 1 |
| OH | 11,374,540 | 17.582 | ***17*** | 18 | 18 | 18 | 18 | 18 |
| OK | 3,458,819 | 5.346 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| OR | 3,428,543 | 5.300 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| PA | 12,300,670 | 19.013 | 19 | 19 | 19 | 19 | 19 | 19 |
| RI | 1,049,662 | 1.622 | 2 | 2 | 2 | 2 | 2 | ***1*** |
| SC | 4,025,061 | 6.222 | 6 | 6 | 6 | 6 | 6 | 6 |
| SD | 756,874 | 1.170 | ***2*** | 1 | 1 | 1 | 1 | 1 |
| TN | 5,700,037 | 8.811 | 9 | 9 | 9 | 9 | 9 | 9 |
| TX | 20,903,994 | 32.312 | ***31*** | 32 | 32 | 32 | 32 | ***33*** |
| UT | 2,236,714 | 3.457 | ***4*** | ***4*** | ***4*** | 3 | 3 | 3 |

| ST | Apportion-ment population | Quota[a] | Apportionment Method: | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Smallest divisors (Adams) | Harm-onic mean (Dean) | Ranked fractional remainders (Hamilton-Vinton) | **Current method: equal pro-portions (Hill)** | Major fractions (Webster) | Largest divisors (Jeffer-son) |
| VT | 609,890 | 0.943 | 1 | 1 | 1 | 1 | 1 | 1 |
| VA | 7,100,702 | 10.976 | 11 | 11 | 11 | 11 | 11 | 11 |
| WA | 5,908,684 | 9.133 | 9 | 9 | 9 | 9 | 9 | 9 |
| WV | 1,813,077 | 2.802 | 3 | 3 | 3 | 3 | 3 | ***2*** |
| WI | 5,371,210 | 8.302 | 8 | 8 | 8 | 8 | 8 | 8 |
| WY | 495,304 | 0.766 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 281,424,177 | | | | | | | |

[a] A state's quota of Representatives is obtained by dividing the population of the fifty states by 435 to obtain a common divisor (645,632 in 2001) which is in turn divided into each state's population.

## Table 3. Seat Assignments in 2001 for Various House Apportionment Formulas (Ranked by State Population)

| ST | Apportion-ment population | Quota[a] | Apportionment Method: | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Smallest divisors (Adams) | Harm-onic mean (Dean) | Ranked fractional remainders (Hamilton-Vinton) | **Current method: equal pro-portions (Hill)** | Major fractions (Webster) | Largest divisors (Jeffer-son) |
| CA | 33,930,798 | 52.450 | ***50*** | ***52*** | ***52*** | 53 | 53 | ***55*** |
| TX | 20,903,994 | 32.312 | ***31*** | 32 | 32 | 32 | 32 | ***33*** |
| NY | 19,004,973 | 29.376 | ***28*** | 29 | 29 | 29 | 29 | ***30*** |
| FL | 16,028,890 | 24.776 | ***24*** | 25 | 25 | 25 | 25 | ***26*** |
| IL | 12,439,042 | 19.227 | 19 | 19 | 19 | 19 | 19 | ***20*** |
| PA | 12,300,670 | 19.013 | 19 | 19 | 19 | 19 | 19 | 19 |
| OH | 11,374,540 | 17.582 | ***17*** | 18 | 18 | 18 | 18 | 18 |
| MI | 9,955,829 | 15.389 | 15 | 15 | 15 | 15 | 15 | ***16*** |
| NJ | 8,424,354 | 13.022 | 13 | 13 | 13 | 13 | 13 | 13 |
| GA | 8,206,975 | 12.686 | 13 | 13 | 13 | 13 | 13 | 13 |
| NC | 8,067,673 | 12.470 | ***12*** | ***12*** | 13 | 13 | 13 | 13 |
| VA | 7,100,702 | 10.976 | 11 | 11 | 11 | 11 | 11 | 11 |
| MA | 6,355,568 | 9.824 | 10 | 10 | 10 | 10 | 10 | 10 |
| IN | 6,090,782 | 9.415 | 9 | 9 | 9 | 9 | 9 | 9 |
| WA | 5,908,684 | 9.133 | 9 | 9 | 9 | 9 | 9 | 9 |
| TN | 5,700,037 | 8.811 | 9 | 9 | 9 | 9 | 9 | 9 |
| MO | 5,606,260 | 8.666 | 9 | 9 | 9 | 9 | 9 | 9 |
| WI | 5,371,210 | 8.302 | 8 | 8 | 8 | 8 | 8 | 8 |
| MD | 5,307,886 | 8.204 | 8 | 8 | 8 | 8 | 8 | 8 |
| AZ | 5,140,683 | 7.946 | 8 | 8 | 8 | 8 | 8 | 8 |
| MN | 4,925,670 | 7.614 | 8 | 8 | 8 | 8 | 8 | ***7*** |
| LA | 4,480,271 | 6.925 | 7 | 7 | 7 | 7 | 7 | 7 |
| AL | 4,461,130 | 6.896 | 7 | 7 | 7 | 7 | 7 | 7 |
| CO | 4,311,882 | 6.665 | 7 | 7 | 7 | 7 | 7 | 7 |
| KY | 4,049,431 | 6.259 | 6 | 6 | 6 | 6 | 6 | 6 |
| SC | 4,025,061 | 6.222 | 6 | 6 | 6 | 6 | 6 | 6 |
| OK | 3,458,819 | 5.346 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| OR | 3,428,543 | 5.300 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| CT | 3,409,535 | 5.270 | ***6*** | 5 | 5 | 5 | 5 | 5 |
| IA | 2,931,923 | 4.532 | 5 | 5 | 5 | 5 | 5 | ***4*** |
| MS | 2,852,927 | 4.410 | ***5*** | 4 | 4 | 4 | 4 | 4 |
| KS | 2,693,824 | 4.164 | 4 | 4 | 4 | 4 | 4 | 4 |
| AR | 2,679,733 | 4.142 | 4 | 4 | 4 | 4 | 4 | 4 |
| UT | 2,236,714 | 3.457 | ***4*** | ***4*** | ***4*** | 3 | 3 | 3 |
| NV | 2,002,032 | 3.095 | 3 | 3 | 3 | 3 | 3 | 3 |
| NM | 1,823,821 | 2.819 | 3 | 3 | 3 | 3 | 3 | ***2*** |
| WV | 1,813,077 | 2.802 | 3 | 3 | 3 | 3 | 3 | ***2*** |
| NE | 1,715,369 | 2.651 | 3 | 3 | 3 | 3 | 3 | ***2*** |
| ID | 1,297,274 | 2.005 | 2 | 2 | 2 | 2 | 2 | 2 |
| ME | 1,277,731 | 1.975 | 2 | 2 | 2 | 2 | 2 | 2 |
| NH | 1,238,415 | 1.914 | 2 | 2 | 2 | 2 | 2 | 2 |
| HI | 1,216,642 | 1.881 | 2 | 2 | 2 | 2 | 2 | ***1*** |

| ST | Apportion-ment population | Quota[a] | Apportionment Method: | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Smallest divisors (Adams) | Harm-onic mean (Dean) | Ranked fractional remainders (Hamilton-Vinton) | **Current method: equal pro-portions (Hill)** | Major fractions (Webster) | Largest divisors (Jeffer-son) |
| RI | 1,049,662 | 1.622 | 2 | 2 | 2 | 2 | 2 | ***1*** |
| MT | 905,316 | 1.399 | ***2*** | ***2*** | 1 | 1 | 1 | 1 |
| DE | 785,068 | 1.213 | ***2*** | 1 | 1 | 1 | 1 | 1 |
| SD | 756,874 | 1.170 | ***2*** | 1 | 1 | 1 | 1 | 1 |
| ND | 643,756 | 0.995 | 1 | 1 | 1 | 1 | 1 | 1 |
| AK | 628,933 | 0.972 | 1 | 1 | 1 | 1 | 1 | 1 |
| VT | 609,890 | 0.943 | 1 | 1 | 1 | 1 | 1 | 1 |
| WY | 495,304 | 0.766 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 281,424,177 | | | | | | | |

[a] A state's quota of Representatives is obtained by dividing the population of the fifty states by 435 to obtain a common divisor (645,632 in 2001) which is in turn divided into each state's population.

## A Framework for Evaluating Apportionment Methods

All the apportionment methods described above arguably have properties that recommend them. Each is the best formula to satisfy certain mathematical measures of fairness, and the proponents of some of them argue that their favorite meets other goals as well. The major issue raised in the Montana case[22] was which formula best approximates the "one person, one vote" principle. The apportionment concerns raised in the Massachusetts case[23] not only raised "one person, one vote" issues, but also suggested that the Hill method discriminates against the larger states.

It is not immediately apparent which of the methods described above is the "fairest" or "most equitable" in the sense of meeting the "one person, one vote" standard. As already noted, no apportionment formula can equalize districts precisely, given the constraints of (1) a fixed size House, (2) a minimum seat allocation of one, and (3) the requirement that districts not cross state lines. The practical question to be answered, therefore, is not how inequality can be eliminated, but how it can be minimized. This question too, however, has no clearly definitive answer, for there is no single established criterion by which to determine the equality or fairness of a method of apportionment.

In a report to the Congress in 1929, the National Academy of Sciences (NAS) defined a series of possible criteria for comparing how well various apportionment formulas achieve equity among states.[24] This report predates the Supreme Court's enunciation of the "one person, one vote" principle by more than 30 years, but if the Congress decided to reevaluate its 1941 choice to adopt the Hill method, it could use one of the NAS criteria of equity as a measure of how well an apportionment formula fulfills that principle.

Although the following are somewhat simplified restatements of the NAS criteria, they succinctly present the question before the Congress if it chose to take up this matter. Which of these measures best approximates the one person, one vote concept?

- The method that minimizes the difference between the largest average district size in the country and the smallest? This criterion leads to the Dean method.

- The method that minimizes the difference in each person's individual share of his or her Representative by subtracting the

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[22] *Department of Commerce* v. *Montana*, 503 U.S. 441 (1992).

[23] *Franklin* v. *Massachusetts*, 505 U.S. 788 (1992).

[24] U.S. Congress, House, Committee on Post Office and Civil Service, Subcommittee on Census and Statistics, *The Decennial Population Census and Congressional Apportionment*, Appendix C: Report of National Academy of Sciences Committee on Apportionment, 91st Cong., 1st Sess., H.Rept. 91-1314 (Washington: GPO, 1970), pp. 19-21.

largest such share for a state from the smallest share? This criterion leads to the Webster method.

- The method that minimizes the difference in average district sizes, or in individual shares of a Representative, when those differences are expressed as percentages? These criteria both lead to the Hill method.

- The method that minimizes the absolute representational surplus among states?[25] This criterion leads to the Adams method.

- The method that minimizes the absolute representational deficiency among states?[26] This criterion leads to the Jefferson method.

In the absence of further information, it is not apparent which criterion (if any) best encompasses the principle of "one person, one vote." Although the NAS report endorsed as its preferred method of apportionment the one currently in use — the Hill method — the report arguably does not make a clear-cut or conclusive case for one method of apportionment as fairest or most equitable. Are there other factors that might provide additional guidance in making such an evaluation? The remaining sections of this report examine three additional possibilities put forward by statisticians: (1) mathematical tests different from those examined in the NAS report; (2) standards of fairness derived from the concept of states' representational

[25] The absolute representational surplus is calculated in the following way. Take the number of Representatives assigned to the state whose average district size is the smallest (the most over represented state). From this number subtract the number of seats assigned to the state with the largest average district size (the most under represented state). Multiply this remainder by the population of the most over represented state divided by the population of the most under represented state. This number is the absolute representational surplus of the state with the smallest average district size as compared to the state with the largest average district size. In equation form this may be stated as follows: $S=(a-b)*(A/B)$ where $S$ is the absolute representation surplus, $A$ is the population of the over represented state, $B$ is the population of the under-represented state, $a$ is the number of representatives of the over represented state, and $b$ is the number of representatives of the under represented state. For further information about this test, see: Schmeckebier, *Congressional Apportionment*, pp. 45-46.

[26] The absolute representational deficiency is calculated in the following way. Take the number of Representatives assigned to the state whose average district size is the largest (the most under represented state). From this number subtract the number of seats assigned to the state with the largest average district size (the most over represented state) multiplied by the population of the under represented state divided by the population of the over represented state. This number is the absolute representational deficiency of the state with the smallest average district size, as compared to the state with the largest average district size. In equation form, this may be stated as follows: $D=b-((a*B)/A)$ where $D$ is the absolute representation deficiency, $A$ is the population of the over represented state, $B$ is the population of the under represented state, $a$ is the number of representatives of the over represented state, and $b$ is the number of representatives of the under represented state. For further information about this test, see Schmeckebier, *Congressional Apportionment*, pp. 52-54.

"quotas"; and (3) the principles of the constitutional "great compromise" between large and small states that resulted in the establishment of a bicameral Congress.

## Alternative Kinds of Tests

As the discussion of the NAS report showed, the NAS tested each of its criteria for evaluating apportionment methods by its effect on pairs of states. (The descriptions of the NAS tests above stated them in terms of the highest and lowest states for each measure, but, in fact, comparisons between all pairs of states were used.) These pairwise tests, however, are not the only means by which different methods of apportionment can be tested against various criteria of fairness.

For example, it is indisputable that, as the state of Montana contended in 1992, the Dean method minimizes absolute differences in state average district populations in the pairwise test. One of the federal government's counter arguments, however, was that the Dean method does not minimize such differences when all states are considered simultaneously. The federal government proposed *variance* as a means of testing apportionment formulas against various criteria of fairness.

The variance of a set of numbers is the sum of the squares of the deviations of the individual values from the mean or average.[27] This measure is a useful way of summarizing the degree to which individual values in a list vary from the average (mean) of all the values in the list. High variances indicate that the values vary greatly; low variances mean the values are similar. If all values in the lists are identical, the variance is zero. According to this test, in other words, the smaller the variance, the more equitable the method of apportionment.

If the variance for a Dean apportionment is compared to that of a Hill apportionment in 1990 (using the difference between district sizes as the criterion), the apportionment variance under Hill's method is smaller than that under Dean's (see **Table 4**). In fact, using average district size as the criterion and variance as the test, the variance under the Hill method is the smallest of any of the apportionment methods discussed in this report.

[27] In order to calculate variance for average district size, first find the ideal size district for the entire country and then subtract that number from each state's average size district. This may result in a positive or negative number. The square of this number eliminates any negative signs. To find the total variance for a state, multiply this number by the total seats assigned to the state. To find the variance for entire country, sum all the state variances.

## Table 4. Alternate Methods for Measuring Equality of District Sizes

| Method | Criteria for evaluation: values to be minimized | | | |
|---|---|---|---|---|
| | Variance | | Sum of absolute values of differences | |
| | Average district size | Individual shares | Average district size | Individual shares |
| Adams | 1,911,209,406 | 0.0354959 | 13,054,869 | 44.2368122 |
| Dean | 681,742,417 | 0.0077953 | 7,170,067 | 22.3962477 |
| Hill (current) | ***661,606,402*** | 0.0058026 | 7,016,021 | 21.3839214 |
| Webster | 665,606,402 | 0.0057587 | 6,997,789 | 21.2530467 |
| Hamilton-Vinton | 676,175,430 | ***0.0057013*** | ***6,977,798*** | ***21.0633312*** |
| Jefferson | 2,070,360,118 | 0.0112808 | 11,149,720 | 31.9326856 |

***Bolded and Italicized*** numbers are the smallest for the category. The closer the values are to zero, the closer the method comes to equalizing district sizes in the entire country. Source: CRS.

Variances can be calculated, however, not only for differences in average district size, but for each of the criteria of fairness used in pairwise tests in the 1929 NAS report. As with those pairwise tests, different apportionment methods are evaluated as most equitable, depending on which measure the variance is calculated for. For example, if the criterion used for comparison is the individual share of a Representative, the Hamilton-Vinton method proves most effective in minimizing inequality, as measured by variance (with Webster the best of the rounding methods).

The federal government in the Massachusetts case also presented another argument to challenge the basis for both the Montana and Massachusetts claims that the Hill method is unconstitutional. It contended that *percent* difference calculations are more fair than absolute differences, because absolute differences are not influenced by whether they are positive or negative in direction.[28]

Tests other than pairwise comparisons and variance can also be applied. For example, Table 4 reports data for each method using the sum of the *absolute values* (rather than the squares) of the differences between national averages and state figures.[29] Using this test for state differences from the national "ideal" both for district sizes and for shares of a Representative, the Hamilton-Vinton method again

[28] *Declaration of Lawrence R. Ernst* filed on behalf of the Government in *Commonwealth of Massachusetts, et. al. v. Mosbacher, et. al.* CV NO. 91-111234 (W.D. Mass. 1991), p. 13.

[29] This is not a "standard" statistical test such as computing the variance. This measure is calculated as follows. Each state's average size district is subtracted from the national "ideal size" district. (In some cases this will result in a negative number, but this calculation uses the "absolute value" of the numbers, which always is expressed as a positive number.) This absolute value for each state is multiplied by the number of seats the method assigns to the state. These state totals of differences from the national ideal size are then summed for the entire nation.

produces the smallest national totals. Of the rounding methods, again, the Webster method minimizes both these differences.

## Fairness and Quota

These examples, in which different methods best satisfy differing tests of a variety of criteria for evaluation, serve to illustrate further the point made earlier, that no single method of apportionment need be unambiguously the most equitable by all measures. Each apportionment method discussed in this report has a rational basis, and for each, there is at least one test according to which it is the most equitable. The question of how the concept of fairness can best be defined, in the context of evaluating an apportionment formula, remains open.

Another approach to this question begins from the observation that, if representation were to be apportioned among the states truly according to population, the fractional remainders would be treated as fractions rather than rounded. Each state would be assigned its exact quota of seats, derived by dividing the national "ideal" size district into the state's apportionment population. There would be no "fractional Representatives," just fractional votes based on the states' quotas.

**Quota Representation.** The Congress could weight each Representative's vote to account for how much his or her constituents were either over or under represented in the House. In this way, the states' exact quotas would be represented, but each Representative's vote would count differently. (This might be an easier solution than trying to apportion seats so they crossed state lines, but it would, however, raise other problems relating to potential inequalities of influence among individual Representatives.[30])

If this "quota representation" defines absolute fairness, then the concept of the quota, rather than some statistical test, can be used as the basis of a simple concept for judging the relative fairness of apportionment methods: a method should never make a seat allocation that differs from a state's exact quota by more than one seat.[31] Unfortunately, this concept is complicated in its application by the constitutional requirement that each state must get one seat regardless of population size. Hence, some modification of the quota concept is needed to account for this requirement.

One solution is the concept of *fair share*, which accounts for entitlements to less than one seat by eliminating them from the calculation of quota. After all, if the Constitution awards a seat for a fraction of less than one, then, by definition, that is the state's fair share of seats.

> To illustrate, consider a hypothetical country with four states having populations 580, 268, 102, and 50 (thousand) and a House of 10 seats to apportion. Then the

[30] For example, Virginia's quota of Representatives based on 2000 Census was 10.976. Based on this quota, each Virginia Representative would be entitled to 1.0976 votes each in the House. Their votes would "weigh" more than Alaska's single Representative whose vote would count 0.972 based on Alaska's quota.

[31] *Fair Representation*, p. 79.

> quotas are 5.80, 2.68, 1.02 and .50. But if each state is entitled to at least one whole seat, then the fair share of the smallest state is 1 exactly. This leaves 9 seats to be divided among the rest. Their quotas of 9 seats are 5.49, 2.54, and .97. Now the last of these is entitled to 1 seat, so its fair share is 1 exactly, leaving 8 seats for the rest. Their quotas of 8 are 5.47 and 2.53. Since these are both greater than 1, they represent the exact fractional representation that these two states are entitled to; i.e. they are the *fair shares*.[32]

Having accounted for the definitional problem of the constitutional minimum of one seat, the revised measure is not the exact quota, but the states' fair shares. Which method meets the goal of not deviating by more than one seat from a state's fair share? *No rounding method meets this test under all circumstances.* Of the methods described in this report, only the Hamilton-Vinton method always stays within one seat of a state's fair share. Some rounding methods are better than others in this respect. Both the Adams and Jefferson methods *nearly always* produce examples of states that get more than one seat above or below their fair shares. Through experimentation we learn that the Dean method tends to violate this concept approximately one percent of the time, while Webster and Hill violate it much less than one percent of the time.[33]

## Implementing the "Great Compromise"

The framers of the Constitution (as noted earlier) created a bicameral Congress in which representation for the states was equal in the Senate and apportioned by population in the House. In the House, the principal means of apportionment is by population, but each state is entitled to one Representative regardless of its population level. Given our understanding that the "great compromise" was struck, in part, in order to balance the interests of the smaller states with those of the larger ones, how well do the various methods of apportionment contribute to this end?

If it is posited that the combination of factors favoring the influence of small states encompassed in the great compromise (equal representation in the Senate, and a one seat minimum in the House) unduly advantages the small states, then compensatory influence could be provided to the large states in an apportionment formula. This approach would suggest the adoption of the Jefferson method because it significantly favors large states.[34]

If it is posited that the influence of the small states is overshadowed by the larger ones (perhaps because the dynamics of the electoral college focus the attention of presidential candidates on larger states, or the increasing number of one-Representative states — from five to seven since 1910), there are several methods

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[32] Balinski, M. L. and H. P. Young, *Evaluation of Apportionment Methods,* Prepared Under a Contract for the Congressional Research Service of the Library of Congress, Contract No. CRS 84-15, Sept. 30, 1984, p. 3.

[33] Ibid., p. 16.

[34] Table 3 rank-orders the states by their 1990 populations. The Jefferson method awards 55 seats to California and 33 seats to Texas when these states' quotas (state population divided by 1/435 of the apportionment population) are 52.45 and 32.31 respectively.

that could reduce the perceived inbalance. The Adams method favors small states in the extreme, Dean much less so, and Hill to a small degree.[35]

If it is posited that an apportionment method should be neutral in its application to the states, two methods may meet this requirement. Both the Webster and Hamilton-Vinton methods are considered to have these properties.[36]

## Conclusion

If Congress decides to revisit the matter of the apportionment formula, this report illustrates that there could be many competing criteria from which it can choose as a basis for decision. Among the competing mathematical tests are the pair-wise measures proposed by the National Academy of Sciences in 1929. The federal government proposed the statistical test of variance as an appropriate means of computing a total for all the districts in the country in the 1992 litigation on this matter. The plaintiffs in Massachusetts argued that variance can be computed for different criteria than those proposed by the federal government — with different variance measures leading to different methods.

The contention that one method or another best implements the "great compromise" is open to much discussion. All of the competing points suggest that Congress would be faced with difficult choices if it decided to take this issue up prior to the 2010 Census. Which of the mathematical tests discussed in this report best approximates the constitutional requirement that Representatives be apportioned among the states according to their respective numbers is, arguably, a matter of judgment — not some indisputable mathematical test.



[35] There is disagreement on this point as it pertains to the Hill method (*Declaration of Lawrence R. Ernst*) but the evidence that the Hill method is slightly biased toward small states is more persuasive than the criticism. See Balinski and Young, *Evaluation of Apportionment Methods*, noted above.

[36] *Evaluation of Apportionment Methods*, p. 10-12.